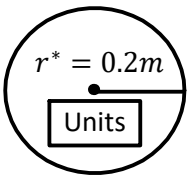
C12 - 4.1 - Shapes Rel Rates Notes

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ | $\frac{dA}{dr} = 2\pi r \times \frac{dr}{dr}$ | $\frac{dr}{dr} = 1$ *

Pebble Dropped in Pond. Find Rate Area Circular Ripple ; Radius is 20cm if Radius Rate is 0.04 m/s.



$\frac{dr}{dt} = 0.04\frac{m}{s}$

$\left.\frac{dA}{dt}\right|_{r=0.2m} = ?$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$ Chain Rule

Leibniz Notation $= 2\pi r \cdot (0.04)$

$= 2\pi(0.2)(0.04)$

$\frac{dA}{dt} = 0.05\frac{m^2}{s}$

Area Rate = $0.05\frac{m^2}{s}$ when r = $0.2m$.

Check Average*

t	r	A
0	0.2	0.1257
1	0.24	0.1809

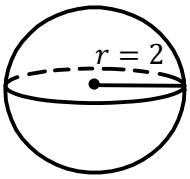
$A = \pi r^2$
$A = \pi(0.2)^2$
$A = 0.1257$
$A = \pi r^2$
$A = \pi(0.24)^2$
$A = 0.1809$

$0.2 + 0.04 = 0.24$

$\frac{0.1809 - 0.1257}{1 - 0} \approx 0.05$

$t = 0.1s^*$

Find Rate of Sphere Radius, R = 2 m, if Volume is Decreasing at $256\frac{m^3}{s}$.



$\frac{dV}{dt} = -256\frac{m}{s^3}$

$-ve$: Shrinking

$\frac{dr}{dt}|_{r=2} = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 3 \times \frac{4}{3}\pi r^{3-1}\frac{dr}{dt}$

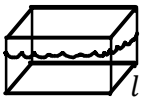
$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$-256 = 4\pi(2)^2\frac{dr}{dt}$

$\frac{dr}{dt} = -\frac{16}{\pi}\frac{m}{s}$

Radius Rate = $-\frac{16}{\pi}\frac{m}{s}$ when R = 2 m.

Water Filling



$w = 60$

$\left.\frac{dh}{dt}\right|_{\frac{dV}{dt}=5\frac{m^3}{s}} = ?$

$V = lwh$

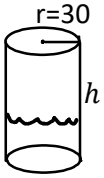
$V = 30(60)h$

$\frac{dV}{dt} = 1800\frac{dh}{dt}$

$5 = 1800\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{360}\frac{m}{s}$

Water Filling



$\left.\frac{dh}{dt}\right|_{\frac{dV}{dt}=500\frac{cm^3}{s}} = ?$

$V = \pi r^2 h$

$V = \pi(30)^2 h$

$V = 900\pi h$

$\frac{dV}{dt} = 900\pi\frac{dh}{dt}$

$500 = 900\pi\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{5}{9\pi}\frac{cm}{s}$

Rectangle's Length Twice Width Perimeter of 40.

$\frac{dP}{dt} = 6\frac{m}{s}$

$\left.\frac{dA}{dt}\right|_{P=40\,m} = ?$

$A = w(2w)$

$A = 2w^2$

$\frac{dA}{dt} = 4w\frac{dw}{dt}$

$\frac{dA}{dt} = 4\left(\frac{20}{3}\right)(1)$

$\frac{dA}{dt} = \frac{80}{3}\frac{m^2}{s}$

$P = 2(2w) + 2(w)$

$P = 6w$

$\frac{dP}{dt} = 6\frac{dw}{dt}$

$6 = 6\frac{dw}{dt}$

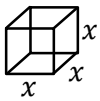
$\frac{dw}{dt} = 1\frac{m}{s}$

$P = 6w$

$40 = 6w$

$w = \frac{20}{3}\,m$

Box Increasing



$\frac{dx}{dt} = 0.5\frac{m^2}{s}$

$\left.\frac{DV}{dt}\right|_{\frac{dSA}{dt}=36\frac{m^2}{s}} = ?$

$SA = 6x^2$

$\frac{dSA}{dt} = 12x\frac{dx}{dt}$

$36 = 12x(0.5)$

$x = 6\,m$

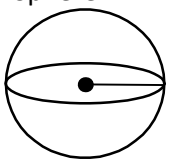
$V = x^3$

$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$

$\frac{dV}{dt} = 3(6)^2(0.5)$

$\frac{dV}{dt} = 54\frac{m^3}{s}$

Sphere



$\left.\frac{DV}{dt}\right|_{SA=100} = ?$

$\frac{dr}{dt} = 2\frac{m}{s}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi\left(\sqrt{\frac{100}{4\pi}}\right)^2\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi \times \frac{100}{4\pi} \times 2$

$\frac{dV}{dt} = 200\frac{m^3}{s}$

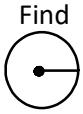
$SA = 4\pi r^2$

$100 = 4\pi r^2$

$r = \sqrt{\frac{100}{4\pi}}$

$r = \frac{10}{2\sqrt{\pi}}\,m$

Find $\frac{dA}{dC_{|C}} = ?$



$C = 2\pi r$

$r = \frac{C}{2\pi}$

$A = \pi r^2$

$A = \pi\left(\frac{C}{2\pi}\right)^2$

$A = \frac{C^2}{4\pi}$

$\frac{dA}{dC} = \frac{1}{4\pi}\,2C\,\frac{dC}{dC}$

$\frac{dA}{dC} = \frac{1}{4\pi}\,2C$

$\frac{dy}{dt} = 6 \quad \left.\frac{dx}{dt}\right|_{y=2} = ?$

$xy^2 = 12$

$\frac{dx}{dt}y^2 + 2y\frac{dy}{dt}x = 0$

$\frac{dx}{dt}(2)^2 + 2(2)(6)(3) = 0$

$\frac{dx}{dt} = -18$

$xy^2 = 12$

$x(2)^2 = 12$

$4x = 12$

$x = 3$